Hong Kong Mathematics Olympiad (2003 – 2004)
Final Event 1 (Individual)
香港数学竞赛 (2003 – 2004)
决赛项目 1 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知有 a 个少于 200 的正整数，它们每个都只有三个正因子，求 a 的值。

Given that there are a positive integers less than 200 and each of them has exactly three positive factors, find the value of a.

2. 若 a 个斜边是 $\sqrt{2}$ cm 的等腰直角三角形能拼成一个周界是 b cm 的梯形，求 b 的最小可能的值。(答案用根号表示)

If a copies of a right-angled isosceles triangle with hypotenuse $\sqrt{2}$ cm can be assembled to form a trapezium with perimeter equal to b cm, find the least possible value of b. (give the answer in surd form)

3. 若 $\sin(c^2-3c+17)^\circ=\dfrac{4}{b-2}$，其中 $0<c^2-3c+17<90$ 及 $c>0$，求 c 的值。

If $\sin(c^2-3c+17)^\circ=\dfrac{4}{b-2}$, where $0<c^2-3c+17<90$ and $c>0$, find the value of c.

4. 已知两个三位数 $\overline{xyz}$ 和 $\overline{zyx}$ 的差等于 $700-c$，其中 $x>z$。若 d 是 $x+z$ 的最大值，求 d 的值。

Given that the difference between two 3-digit numbers $\overline{xyz}$ and $\overline{zyx}$ is $700-c$, where $x>z$. If d is the greatest value of $x+z$, find the value of d.

Hong Kong Mathematics Olympiad (2003 – 2004)
Final Event 2 (Individual)
香港数学竞赛 (2003 – 2004)
决赛项目 2 (个人)

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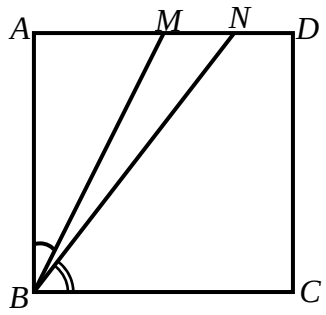
1.



图一
Figure 1

如图一，$ABCD$ 为一正方形，M 是 AD 的中点及 N 是 MD 的中点。若 $\angle CBN : \angle MBA = P : 1$，求 P 的值。

In Figure 1 , $ABCD$ is a square , M is the mid-point of AD and N is the mid-point of MD . If $\angle CBN : \angle MBA = P : 1$, find the value of P .

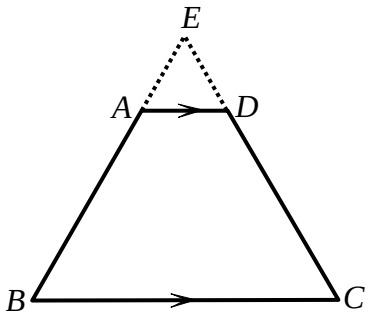
2. 已知 $ABCD$ 为一坐标平面上的菱形，其顶点的坐标分别为 $A(0, 0)$，$B(P, 1)$， $C(u, v)$ 及 $D(1, P)$。若 $u + v = Q$，求 Q 的值。

Given that $ABCD$ is a rhombus on a Cartesian plane, and the co-ordinates of its vertices are $A(0, 0)$, $B(P, 1)$, $C(u, v)$ and $D(1, P)$ respectively. If $u + v = Q$, find the value of Q .

3. 若 $1 + (1 + 2) + (1 + 2 + 3) + \cdots + (1 + 2 + 3 + \cdots + Q) = R$，求 R 的值。

If $1 + (1 + 2) + (1 + 2 + 3) + \cdots + (1 + 2 + 3 + \cdots + Q) = R$, find the value of R .

4.



图二

Figure 2

如图二， EBC 是一等边三角形， A 和 D 分别在 EB 和 EC 上。已知 $AD // BC$ ， $AB = CD = R$ ，且 $AC \perp BD$ 。若梯形 $ABCD$ 的面积是 S，求 S 的值。

In Figure 2, EBC is an equilateral triangle, and A , D lie on EB and EC respectively . Given that $AD // BC$, $AB = CD = R$ and $AC \perp BD$. If the area of the trapezium $ABCD$ is S , find the value of S .

Hong Kong Mathematics Olympiad (2003 – 2004)
Final Event 3 (Individual)
香港数学竞赛 (2003 – 2004)
决赛项目 3 (个人)

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1. 设 $x \neq \pm 1$ 及 $x \neq -3$。若 a 是方程 $\frac{1}{x-1}+\frac{1}{x+3}=\frac{2}{x^2-1}$，求 a 的值。

Let $x \neq \pm 1$ and $x \neq -3$. If a is the real root of the equation $\frac{1}{x-1}+\frac{1}{x+3}=\frac{2}{x^2-1}$, find the value of a.

2. 设 $b>1$，$f(b)=\frac{-a}{\log_2 b}$ 及 $g(b)=1+\frac{1}{\log_3 b}$。若 b 满足方程 $|f(b)-g(b)|+f(b)+g(x)=3$，求 b 的值。

Let $b>1$, $f(b)=\frac{-a}{\log_2 b}$ and $g(b)=1+\frac{1}{\log_3 b}$. If b satisfies the equation $|f(b)-g(b)|+f(b)+g(x)=3$, find the value of b.

3. 已知实数 x_0 满足方程 $x^2-5x+(b-8)=0$。若 $c=\frac{{x_0}^2}{{x_0}^4+{x_0}^2+1}$，求 c 的值。

Given that x_0 satisfies the equation $x^2-5x+(b-8)=0$. If $c=\frac{{x_0}^2}{{x_0}^4+{x_0}^2+1}$, find the value of c.

4. 若 -2 和 $216c$ 是方程 $px^2+dx=1$ 的根，求 d 的值。

If -2 and $216c$ are the roots of the equation $px^2+dx=1$, find the value of d.

Hong Kong Mathematics Olympiad (2003 – 2004)
Final Event 4 (Individual)
香港数学竞赛 (2003 – 2004)
决赛项目 4 (个人)

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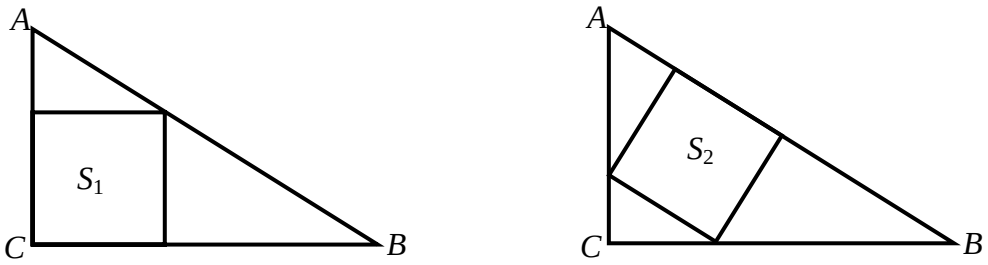
1. 设 a 为实数。若 a 满足方程 $\log_2(4^x+4)=x+\log_2(2^{x+1}-3)$，求 a 的值。

Let a be a real number . If a satisfies the equation $\log_2(4^x+4)=x+\log_2(2^{x+1}-3)$, find the value of a .

2. 已知 n 是自然数。若 $b=n^3-4an^2-12n+144$ 是质数，求 b 的值。

Given that n is a natural number . If $b=n^3-4an^2-12n+144$ is a prime number , find the value of b .

3.



图一
Figure 1

如图一， S_1 和 S_2 都是直角三角形 ABC 的两个不同的内接正方形。若 S_1 的面积是 $40b+1$，S_2 的面积是 $40b$，及 $AC+CB=c$，求 c 的值。

In Figure 1 , S_1 and S_2 are two different inscribed squares of the right-angled triangle ABC . If the area of S_1 is $40b+1$, the area of S_2 is $40b$ and $AC+CB=c$, find the value of c .

4. 已知 $241c+241=d^2$，求 d 的值。

Given that $241c+241=d^2$, find the value of d .